

PROBABILITY PROBLEMS: PERSONAL-OVERALL

MAY 2019

1. Let  $\{S_n\}$  be a simple random walk in one dimension, with  $S_0 = 0$ , and let

$$\tau = \tau_{[0,5]^c} = \inf\{n : S_n \notin [0, 5]\}$$

be the first time the random walk exits the set  $\{0, 1, 2, 3, 4, 5\}$ . Evaluate  $\mathbb{E}[S_{\tau-1}]$   
(Hint: use Wald's identity)

2. Prove that for any two events,

$$|\mathbf{P}\{A \cap B\} - \mathbf{P}\{A\}\mathbf{P}\{B\}| \leq \frac{1}{4}$$

and

$$\mathbf{P}\{A \cup B\}\mathbf{P}\{A \cap B\} \leq \mathbf{P}\{A\}\mathbf{P}\{B\}.$$